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4. FREE ELECTRON THEORY OF METALS & TRANSPORT PROPERTIES

1. The out standing Properties of metals

- Metals are the good electrical and thermal conductivity.

i.e High electrical and thermal conductivities

- In accordance with the Ohm's law, The current density in the steady state is proportional to applied field strength

$$I \propto E$$

$$IR = E$$

- The specific resistivity of metals at room temperature is of the order of $10^{-5} \Omega \cdot cm$

$$\rho = RA/l$$

- Above the Debye temp. the resistivity of metals is proportional to absolute temperature.

$$\rho \propto T.$$

- At low temperature but above approximately $20^\circ K$ the resistivity of many metals is proportional to fifth power of absolute temperature

$$\rho \propto T^5$$

- For most of metals, the resistivity is inversely proportional to pressure

$$\rho \propto \frac{1}{P}$$

- Metal satisfies Widemann-Frantz relation which states that, the ratio of thermal conductivity to the electrical conductivity is proportional to absolute temperature

i.e $\frac{K}{\sigma} \propto T$

where K - thermal conductivity
σ - Electrical conductivity

- A number of metals exhibits the phenomenon of superconductivity i.e Their resistivity disappear at a temperature above absolute zero

$T \to 0K \cdot \rho \to 0$

- The conductivity of metals varies in the presence of magnetic field. This effect is known as magnetoresistance.

- The resistivity of impure specimen given by Mathiessan's rule

$$\rho = \rho_0 + \rho(T)$$

where ρ_0 is constant for impure specimen and $\rho(T)$ is temperature dependant resistivity of the pure specimen.

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1. Drude-Lorentz Classical theory

The theory of electrical conductivity of metals was developed by Paul Drude in 1900 and was modified by Lorentz.

According to Drude metals are composed of positive metal ions whose valance electrons are free to move in it with only the restriction is that the electrons moves ~~on~~ within boundry of metals. The metal ions are bounded to the electrons by an electrostatic attraction between their positive charges and negatively charge electrons. The ~~parent~~ atoms of metals continously dissociating into negative electrons and these electrons are free to move in all direction within the metals. When electric field is applied to metals the negative electrons drift towards positive field direction and produce current in the metal. To prevent the electrons from indefinately acceleration it is assumed that, the collide elastically with metal ions this leads to the steady state current which is proportional to an applied voltage and explain the origin of ohm's law.

Parent

After Drude, Lorentz carried this model and come to close by applying Maxwell-Boltzman statistics to electron gas. The mutual repulsion between negatively charge electrons was neglected

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and potential field due to positive ions assume to be constant everywhere.

The basic postulates on which the theory is based.

1. The positive ions are fixed in position
2. The electrons in metals are free to move. The motion of electrons is limited, it means electrons move within the boundary of metals.
3. In the absence of electric field the electrons follows Maxwell-Boltzman statistics.

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Electrical Conductivity

When electric field E is subjected to the electrons of charge $-e$ and mass m, it produce acceleration a, then equation of motion for the electron in electric field E is

$$ma = -eE \quad \text{------------} (1)$$

$$m\frac{d^2x}{dt^2} = -eE$$

$$\frac{d^2x}{dt^2} = -\frac{e}{m}E$$

Integrating we get

$$\text{velocity} = \frac{dx}{dt} = -\frac{e}{m}Et + C \quad \text{------} (2)$$

If free time or time taken between two successive collisions be τ and velocity along one direction be u then

$$\tau = \frac{\lambda}{u} \quad \text{------------} (3)$$

where λ is mean free path

At $t=0$, $\frac{dx}{dt} = 0$

Apply this condition to equation (2) we get

$$C = 0$$

Substitute this value in equation (2)

$$v = \frac{dx}{dt} = -\frac{e}{m}E + 0$$

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Therefore, average velocity between two successive collision

$$\left(\frac{dx}{dt}\right)_{ave} = \bar{v} = \frac{1}{\tau}\int_0^{\tau} -\frac{e}{m}E\,t\,dt$$

$$= -\frac{1}{\tau}\frac{e}{m}E\left[\frac{t^2}{2}\right]_0^{\tau}$$

$$= -\frac{eE}{m}\frac{1}{\tau}\cdot\frac{\tau^2}{2}$$

$$\bar{v} = -\frac{eE}{m}\cdot\frac{\tau}{2} \quad \text{———} (4)$$

If i is the current density and n number of electrons per unit vol. then we have

$$i = -ne\bar{v}$$

using eqn. (4) in above we have

$$i = -ne - \frac{eE}{m}\cdot\frac{\tau}{2}$$

$$= \frac{1}{2}\frac{ne^2E\tau}{m}$$

$$\because \tau = \frac{\lambda}{u}$$

$$= \frac{1}{2}\frac{ne^2E}{m}\cdot\frac{\lambda}{u}$$

multiply and devided by u we ge

$$i = \frac{1}{2}\frac{ne^2E\lambda u}{mu^2} \quad \text{———} (5)$$

According to kinetic theory we know that

$$\frac{1}{2}mu^2 = \frac{3}{2}KT$$

$$mu^2 = 3KT \quad \text{---------} (6)$$

substitute this value of mu^2 from equation (6) in equation (5)

$$i = \frac{1}{2}\frac{ne^2E\lambda u}{3KT}$$

$$= \frac{ne^2E\lambda u}{6KT}$$

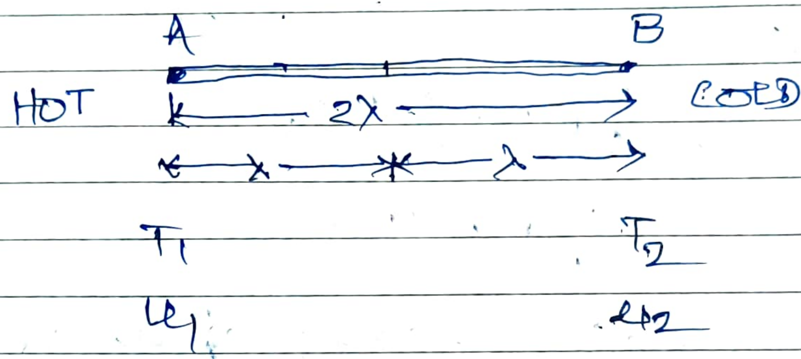
or $i = \sigma E$ $\quad \therefore \sigma = \frac{ne^2\lambda u}{6KT}$ --- (7)

$i \propto E$ which is ohm's law

where σ is electrical conductivity

Thermal conductivity (K)

To obtain the expression for thermal conductivity, consider the two points A & B separated by the distance 2λ



On the basis of kinetic theory, If two Temperatures of T_1 & T_2 are equals i.e $T_1 = T_2$ then there is no transfer of heat energy.

If $T_1 > T_2$ then transfer of heat energy from A to B.

The no. of electrons per unit area per unit time is $\frac{1}{6} nu$, where n is the density of electron and u is their velocity

The energy carried by electron starting from A is kinetic energy i.e $\frac{1}{2} m u_1^2$ where u_1 is the velocity at point A.

∴ At point A

$$\frac{1}{2} m u_1^2 = \frac{3}{2} K T_1 \quad \text{——— (1)}$$

Therefore, total energy transfer from A to B

$$= \frac{1}{6} nu \cdot \frac{1}{2} m u_1^2$$

$= \frac{1}{\cancel{6}_2} nu \cdot \frac{\cancel{3}}{2} kT_1$

$= \frac{1}{4} nukT_1$ ——— (2)

Similarly the energy transfer from from B to A may be calculated as

$\frac{1}{2\cancel{6}} nu \cdot \frac{\cancel{3}}{2} kT_2$

$= \frac{1}{4} nukT_2$ ——— (3)

∴ Net energy transfer from A to B is

$\nabla E = \frac{1}{4} nukT_1 - \frac{1}{4} nukT_2$

$= \frac{1}{4} nuk[T_1 - T_2]$ ——— (4)

If K is the thermal conductivity then the energy transfer per unit are per unit time is or The energy transfer may be expressed interms of thermal conductivity K

$\nabla E = K \cdot$ Temp. gradient

$= K\left[\frac{T_1 - T_2}{2\lambda}\right]$ ——— (5)

equating equation (4) & (5) we get

$K\left[\frac{T_1 - T_2}{2\lambda}\right] = \frac{1}{4} nuk[T_1 - T_2]$

$$K = \frac{\frac{1}{4} n u k \cancel{[T_1 - T_2]} \cdot 2\lambda}{\cancel{[T_1 - T_2]}}$$

$$= \frac{1}{\cancel{4}\,2} n u k \cdot \cancel{2}\lambda$$

$$K = \frac{n u k \lambda}{2} \tag{6}$$

is the expression for thermal conductivity.

Wiedemann - Franz law

It states that, The ratio between thermal conductivity to the electrical conductivity is proportional to temperature (T)

$\frac{\text{Thermal conductivity}}{\text{Electrical conductivity}}$ is proportional to temp. T

$$\frac{K}{\sigma} \propto T$$

We know the equation for electrical conductivity from previous article

$$\therefore \frac{K}{\sigma} = \frac{\frac{nuk\lambda}{2}}{\frac{ne^2\lambda u}{3kT}} = \frac{\cancel{n}\cancel{u}k\cancel{\lambda}}{2} \times \frac{3kT}{\cancel{n}e^2\cancel{\lambda}\cancel{u}}$$

$$\frac{K}{\sigma} = 3\left(\frac{k}{e}\right)^2 T \qquad \text{——— (7)}$$

i.e $\frac{K}{\sigma} \propto T$

or $\frac{K}{\sigma T} = L$ where L is Lorenz number, This is Wiedmann-Franz relation

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$$L = \frac{K}{\sigma T}$$

$T = 20^\circ C = 20 + 273 = \underline{293\,K}$

$K = 386\, W m^{-1} K^{-1}$

$\rho = 1.72 \times 10^{-8}\ \Omega \cdot m$

$$\sigma = \frac{1}{\rho} = \frac{1}{1.72 \times 10^{-8}}$$

$$= 0.5813 \times 10^{8}$$

$$= 5.813 \times 10^{7}$$

$$L = \frac{386}{5.813 \times 10^{7} \times 293}$$

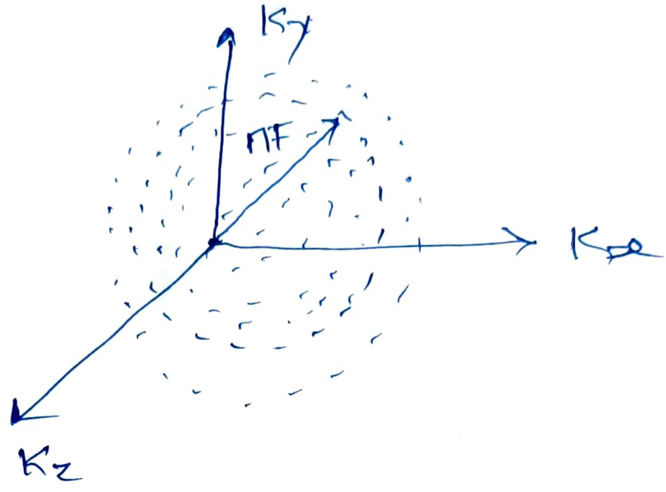
$$= \frac{386}{1702.33} \times 10^{-7}$$

$$= 0.2266 \times 10^{-8}\ \underline{W \Omega K^{-2}}$$

Fermi-Energy

Suppose there are N non interacting electrons contained in the box at absolute zero temp.

At $T = 0°K$, all the level below certain level will be filled while all above it will be empty. The level which devide filled level and vacant level is known as Fermi-level at $T = 0°K$, and it is denoted by $E_F(0)$. Now we shall express Fermi-level as a function of e^- conc. As there are two independent state corresponding to spin orientation. If we draw the sphere the radius is equal to 'n_F' in this sphere we choose the radius such that the point represent occupied energy level there is points which are out side the sphere which is not filled on the point on the circle that gives the Fermi-energy. Therefore Fermi energy is the is the energy of surface of sphere



Fermi sphere

We know that

$$K^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{\hbar^2 K^2}{2m}$$

The Fermi energy on the surface of sphere whose radius is n_F

$$E_F = \frac{\hbar^2}{2m} \cdot K^2 n_F^2 \quad ----(1)$$

As there are two independent state to hold NL^3 electrons at absolute zero temp., then

$2 \times$ vol. of sphere

$$2 \times \frac{4\pi}{3} n_F^3$$

$$\therefore NL^3 = 2 \times \frac{4\pi}{3} n_F^3$$

$\cancel{N'}$

$$n_F^3 = \frac{3N}{8\pi} L^3$$

$$n_F^2 = \left(\frac{3N}{8\pi}\right)^{2/3} \cdot L^2 \quad \text{------(2)}$$

$\therefore$ The energy corresponds to n_F

$$E_F = \frac{\hbar^2}{2m} \cdot K^2 \cdot n_F^2$$

the component of wave vector has the form $K = \frac{2\pi}{L}$

$$\therefore \quad E_F = \frac{\hbar^2}{2m} \cdot \left(\frac{2\pi}{L}\right)^2 \cdot \left(\frac{3N}{8\pi}\right)^{2/3} \cdot L^2$$

$$= \frac{\hbar^2}{2m} \cdot \frac{\cancel{4}\pi^2}{\cancel{L^2}} \cdot \frac{(3N)^{2/3}}{(\cancel{2^3})^{2/3} \cdot \pi^{2/3}} \cdot \cancel{L^2}$$

$$= \frac{\hbar^2}{2m} (3N)^{2/3} \pi^2 \cdot \pi^{-2/3}$$

$$E_F = \frac{\hbar^2}{2m} \left[3N\pi^2\right]^{2/3} \quad \text{---(3)}$$

$\therefore \pi^{\frac{2}{1} - 2/3}$

$\pi^{\frac{6-2}{3}} = \pi^{4/3}$

$(\pi^2)^{2/3}$

This equation shows the Fermi-energy and Fermi-energy may be calculated by knowing the electron concentration

The observed value of Fermi-energy for no. of metal is in eV

metal	Fermi energy in eV
Rb	
K	1.82
Na	3.12
Li	4.72
Cu	7.07
Al	11.7

Enrico - Fermi
Italian Physicist
called Architect of Nuclear age
and Architect of Nuclear bomb
work - theoretical & Experimental
physicist
born - itali (Roam)
died - USA
Award - N. P 1938
(Research - existance of new
radioactive elements produced
by neutron irradiation)
i.e bombardment of element to
produced Fissionable istopes

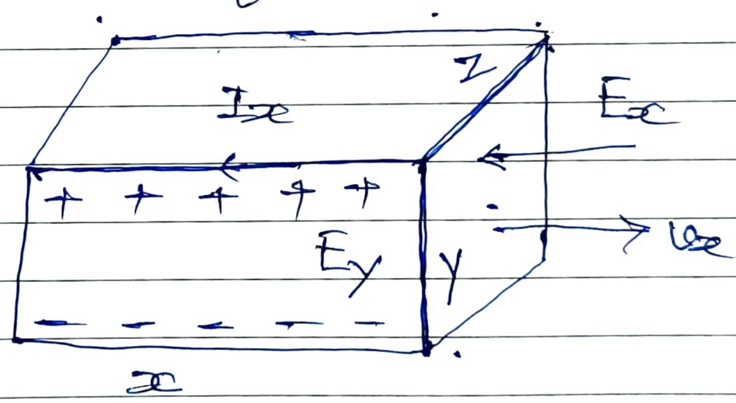
- creator of Ist Nuclear reactor
- Several patent related to
use of nuclear power
- contribution
Statistical Mech, Quantum Mech
Nuclear & Particle Physics
Known For.
Fermi Dirac statics

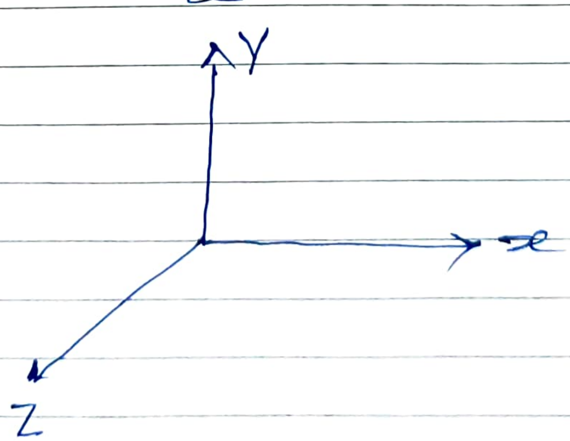
Highest occupied level by electrons
at absolute zero.

Hall effect (1879)

If a magnetic field is applied perpendicular to the conductor carrying current a voltage is developed across the conductor in a direction perpendicular to the both current and magnetic field. This is known as Hall effect.

If we assume a slab of material subjected to an external electric field E_x along x axis as shown in the figure. Because of applied current density I_x will be flow in the direction of E_x.





Electric current flow through the conductor due to flow of electrons. So such a charge particles while moving through the conductor. If we apply magnetic

Field perpendicular to the direction of electric field. So the moving electrons experience a Force. This Force is known as Lorentz Force. This force work in a perpendicular direction to both electric and magnetic field. Due to this Lorentz Force charge particles deflected towards the surface of conductor but they can not escape from the conductor and hence developed electric field Ey.

Hall effect are useful for determination of charge carriers. Also it is useful in the study of semiconductor. Hall effect also useful is studying the density of charge carrier.

Edwin Hall

(7 Nov, 1855 – 20 Nov, 1938) USA

- Research in – thermoelectricity at Harvard Univ. cambridge – 1636 Estd.

written – Numerous text books & Lab manuals.

- 1880 – Ph.D. John Hopkin university

Hall effect (1879) – doctoral thesis in physics.

(161 N.P. Alumni in Harvard)

maryland, Baltimore

Material Science

condensed matter / SSP

1. – charge carrier sign (hole & electron) is determine
2. carrier conc. n / (no. of charge carrier per unit vol.) determine
3. mobility of charge carrier is determine directly
4. we can decided whether material is S.C, metal, S.C & insulator

importance of Hall effect –

Hall voltage and Hall coefficient

To calculate Hall voltage and Hall coeffi-cient, we consider the force acting on electron of charge $-e$ due to electric and magnetic field

Force acting on the electron due to electric field $-eE$

The Force acting on the electron due to magnetic field is

$$\frac{e}{c}[V \times H]$$

Total force acting on the electrons due to combination of electric and magnetic field is

$$E = \cancel{F} - eE + \frac{e}{c}[V \times H]$$

In the present case above equa-tion can be written as

$$F_y = -eE_y + \frac{e}{c}[V_x \times H_z] \quad \text{——— (1)}$$

In the steady state $F_y = 0$

$$0 = -eE_y + \frac{e}{c}[V_x \times H_z]$$

$$\cancel{e} E_y = \frac{\cancel{e}}{c}[V_x \times H_z]$$

$$E_y = \frac{1}{c}[V_x \times H_z] \quad \text{——— (2)}$$

where E_y is the Hall voltage

or Hall field and V_x is the average drift velocity of electrons.

If n is the number of electrons per unit volume, the current density is given by

$$I_x = -neV_x \quad \text{———} (3)$$

$$V_x = -\frac{I_x}{ne} \quad \text{———} (4)$$

using equation (4) in (2)

$$E_y = \frac{1}{c}\left[-\frac{I_x}{ne} \times H_z\right]$$

or $$\frac{E_y}{I_x H_z} = -\frac{1}{nec} = R_H \quad \text{———} (5)$$

where R_H is Hall coefficient for the substance. From the above equation it can be conclude that lower the carrier concentration higher the magnitude of Hall coefficient. The negative value of R_H indicates that the current carriers are electrons. If the charge carriers are holes the R_H i.e. Hall coefficient is positive.

Mobility & Hall angle.

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If a particle carrying current acquired velocity, then velocity per unit electric field is known as mobility and is denoted by μ

$$\mu = V_x / E_x$$

$$V_x = \mu E_x \quad \text{——— (1)}$$

we know that

$$E_y = \frac{1}{c}[V_x \times H_z] \quad \text{——— (2)}$$

$$= \frac{1}{c}[\mu E_x \times H_z]$$

$$E_y = \frac{1}{c} \mu E_x \times H_z \quad \text{——— (3)}$$

Also we know $\frac{E_y}{I_x H_z} = R_H$

$$\text{or } E_y = R_H I_x H_z \quad \text{——— (4)}$$

compairing equation (3) & (4)

$$\frac{1}{c} \mu E_x H_z = R_H I_x H_z$$

$$\mu = \frac{R_H I_x \cancel{H_z} c}{E_x \cancel{H_z}}$$

$$\mu = \frac{R_H I_x c}{E_x} \quad \text{——— (5)}$$

$E = IR$

$R = \frac{E}{I}$

$\frac{1}{R} = \sigma = \frac{I}{E}$

$\sigma = \frac{I_x}{E_x}$

using above equation (5) becomes

$\mu = R_H \sigma C$ ——— (6) in esu

and emu $\mu = R_H \sigma$ ——— (7)

$\mu = \frac{E_y}{I_x H_z} \cdot \sigma$ ----- (8)

Also we know that $I_x \propto E_x$

$I_x = \sigma E_x$

then equation (8) becomes

$\mu = \frac{E_y \cancel{\sigma}}{\cancel{\sigma} E_x H_z}$

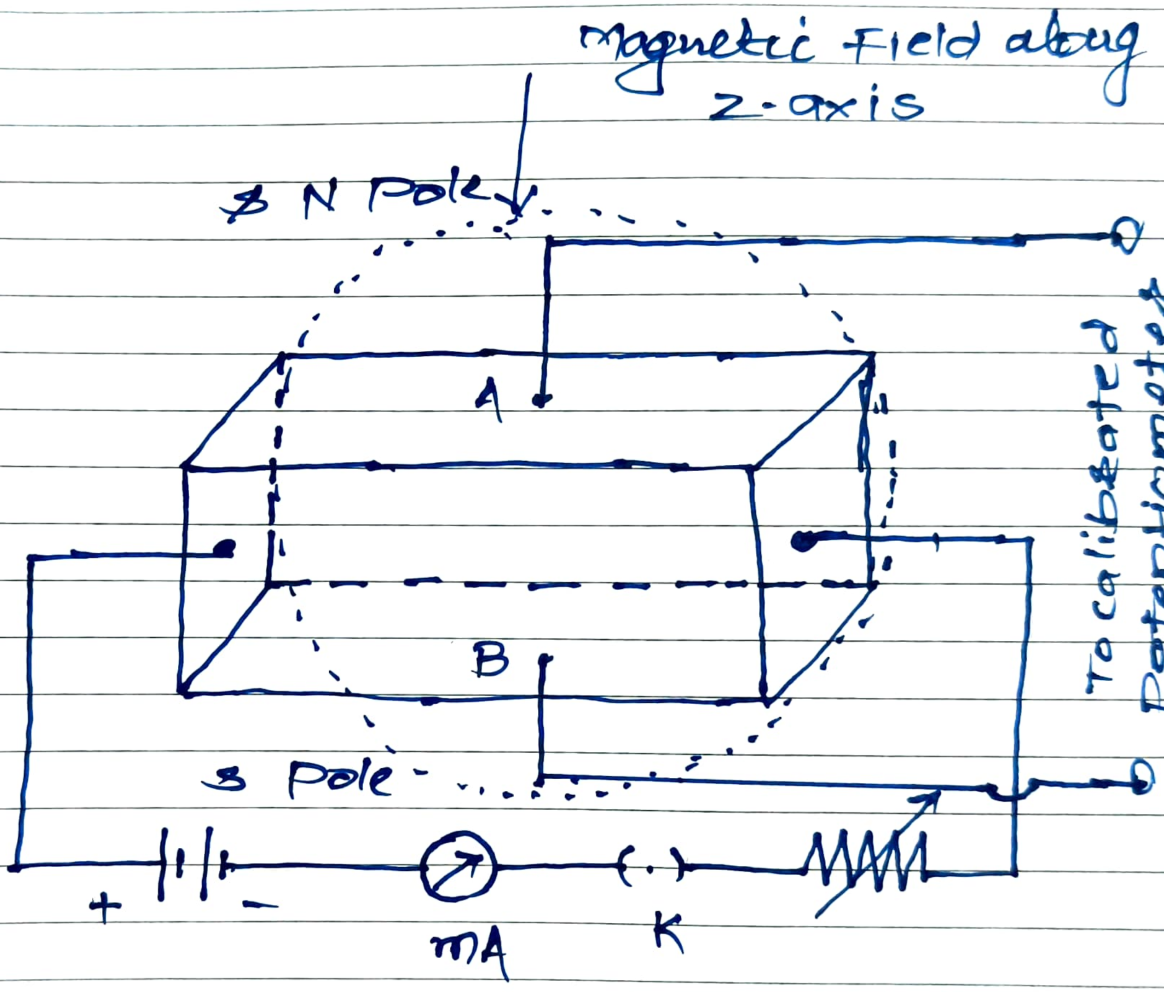
$\therefore \mu = \frac{E_y}{E_x} \cdot \frac{1}{H_z}$

$\mu = \theta_H \cdot \frac{1}{H_z}$

where $\theta_H = \frac{E_y}{E_x}$ is the Hall angle

$\theta_H = \mu \cdot H_z$ ——— (9)

Experimental determination of Hall effect



A thin metallic strip a several mm wide and several cm long is place in x direction and magnetic field H_z is applied in z direction.

A suitable current is passed through the specimen which can be adjusted by the rheostate. The two potential leads are placed between the points A & B which is connected to sensitive calibrated potentiometer to measure the developed Hall voltage V_H.

By measuring Hall voltage Hall coefficient can be calculated as given below.

The value of Hall electric field is given by

$$E_y = -\frac{1}{c}\frac{I_x}{ne}H_z \quad \text{-----}(1)$$

Let b and d be the breath and thickness of the specimen respectively

The current density is given by

$$I_x = \frac{\text{Current}}{\text{Area of speciment}}$$

$$= \frac{i}{b\times d} \quad \text{-----}(2)$$

substitute this value in equation 1 we get.

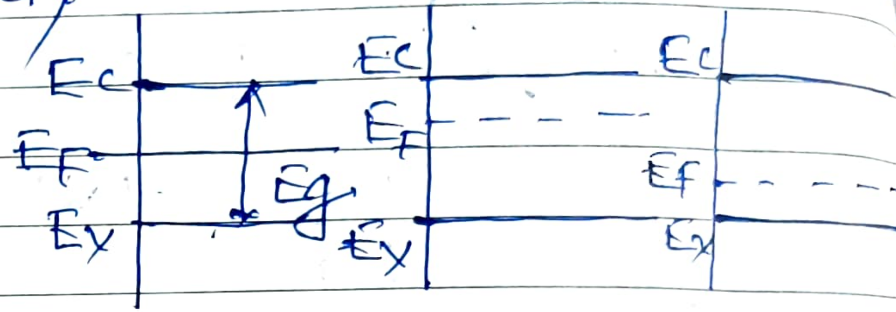
$$E_y = -\frac{1}{c}\frac{iH_z}{nebd}$$

or $$E_y d = V_H = -\frac{1}{c}\frac{iH_z}{neb}$$

$$E_y d = V_H = R_H \frac{iH_z}{b}$$

$$\therefore R_H = -\frac{1}{nec}$$

Fermi level in
- intrinsic semiconductor
exact middle way
N. type S.C.
near to C.B.
P. Type S.C
near to V.B



$E_{F(0)}$

$$K^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{\hbar^2 K^2}{2m}$$

$\hbar = h/2\pi$

$\hbar = h/2\pi$

$$\therefore R_H = \frac{V_H \cdot b}{i H_z} \quad \text{------ (3)}$$

Thus, the value of Hall coefficient R_H can be calculated by knowing i & H_z and by measuring Hall voltage V_H developed across the point A and B by means of callibrated potentiometer.

Important of Hall effect

The Hall effect measurements provided the following information about the solids

1. Sign of charge carriers is determined (electrons or holes)

2. carrier concentration i.e no. of charge carriers per unit volume is determined

3. Mobility of charge carriers is measured directly

4. one can determined whether the material is a metal, semiconductor or insulators

5. From the knowledge of measured Hall voltage, the unknown magnetic field can be ~~determined~~ measured provided the value of Hall constant for the material slab is known.